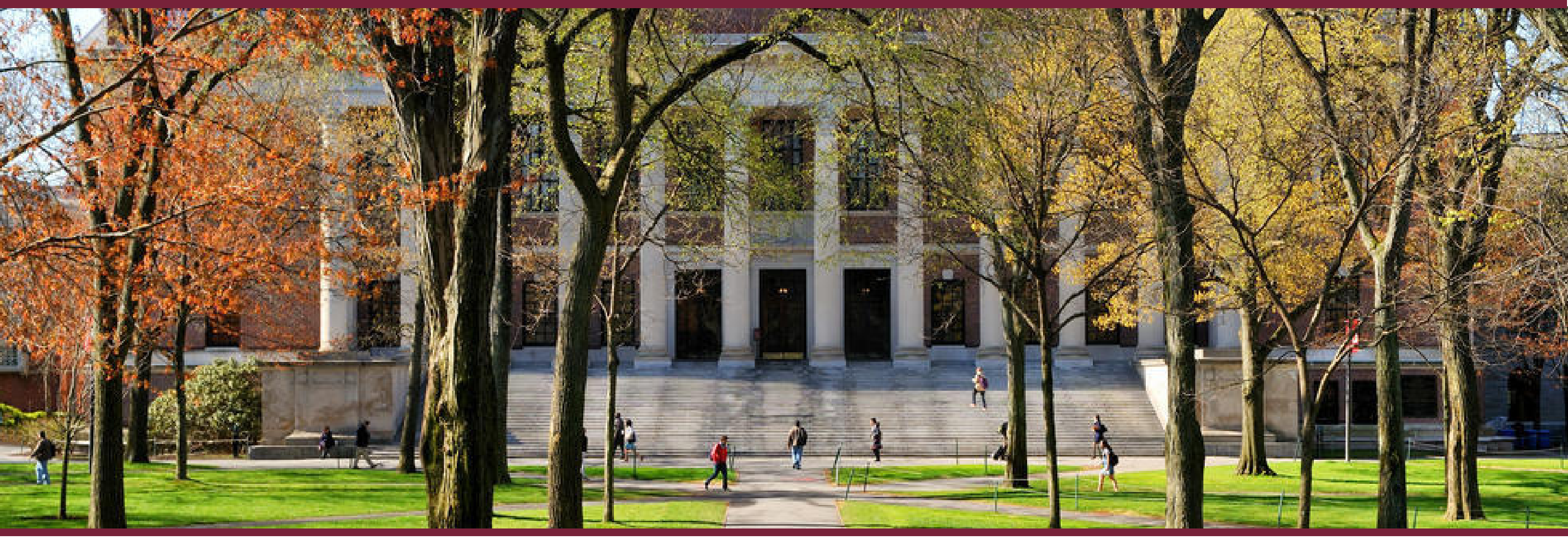
COVID-19 Rapid Response Impact Initiative | White Paper 6

Why We Must Test Millions a Day

April 8, 2020



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Abstract

There is growing consensus from leading think tanks such as the American Enterprise Institute and the Center for American Progress that the way out of lockdown is through a massive testing and tracing infrastructure. Yet there is much less clarity on how large this infrastructure must be to allow a safe return to work. Both the AEI and CAP proposals suggest that hundreds of thousands of tests per day might suffice. However, to date, we are not aware of epidemiological models that attempt to estimate the scale of required testing. This paper tries to fill this gap with rough and preliminary but easily explicable calculations. These suggest that, depending on what tracing technology is used in conjunction with testing, at least millions and possibly hundreds of millions of tests per day will be needed. While we estimate that such capacity is possible by late spring or early summer, we believe that the AEI and CAP timetables and cost estimates are unrealistic and that we must invest much more aggressively if we are to allow a return to work.

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01 Introduction

In the last week, a consensus has emerged qualitatively around a framework for getting back to work that Allen et al. (2020a, 2020b) call “Mobilize and Transition” and others have called “The Hammer and the Dance.” In this framework, massive investments in testing-and-tracing capacity in the very near term (i.e., the next few months) allow a (perhaps gradual) transition back to a relaxed set of restrictions on social interactions that would enable most of the population to go back to work. Both the American Enterprise Institute (AEI) and the Center for American Progress (CAP) have embraced variants on this approach.

This approach immediately raises the question of how large the required testing infrastructure must be to permit a reasonable expectation of success. Unfortunately, to our knowledge, no epidemiological analysis to date provides clear guidance on this. This creates a serious risk that Congress, which will shortly decide on funding for testing, will dramatically overshoot—or worse, undershoot—the investment required to allow a return to work. This could easily set back the timetable for returning to work by months, and thus cost the economy a trillion dollars or more to save, at most, hundreds of millions of dollars. This would be the ultimate false economy.

To avoid such an outcome, this paper attempts a simple, but also clearly explicable, quantitative analysis of the requisite testing capacity under different scenarios to estimate the supporting investment in tracing infrastructure required to accompany the testing. We supplement these informal and transparent calculations with an appendix containing more detailed formal modeling, which use different modeling approaches but arrive at roughly similar conclusion—two in particular.

First, estimates from the AEI and CAP are low, depending on the scenario used, by 1 to 3 orders of magnitude. Even under the most optimistic scenarios, we need to be testing millions of people per

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day to allow a significant return to the workforce. Tens of million per day seems more likely and more than 100 million may be necessary in the worst case. The basic logic here is extremely simple. If we are to use random testing to control the growth of the disease, we must on average catch cases before they spread to more than one other person. The disease spreads on average, when uncontrolled, once every six days and the test has roughly a 20% false negative rate, meaning it will not catch all cases. Thus, we must test the entire population roughly once every three to four days to control the disease, which would require almost a hundred of millions of tests a day. Targeting can of course improve this testing scenario, but this calculation gives a sense of the magnitude of testing required.

Second, tracing capacity makes orders of magnitude difference in the requisite investment, as illustrated in greater mathematical detail in the first appendix model. While 1 to 10 million tests per day may suffice if we have in place a precise tracing infrastructure, such as that provided by high-prevalence and high-reliability Bluetooth-based apps or large scale manual tracing, purely random or even geographically targeted testing could require more than 100 million tests per day. The likeliest scenarios, in which a hodgepodge of different apps and regimes mix and achieve reasonable but imprecise targeting, make a target of tens of millions of tests per day a good focal estimate.

Targeting tests by tracing infections has an outsize impact because it allows us to make much better use of patients who come in for testing due to symptom onset. In fact, given our parameter values, the fact that people with symptoms will come in for testing yields no improvement at all in the number of random tests that must be administered. Symptoms usually start about a third of the way into the infection, at roughly day 5 (Li et al. 2020; Zhang et al. 2020), when the patient will have already spread the infection. This means that simply testing those with symptoms and quarantining them cannot suffice to control the disease.

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However, in this situation the typical person to whom the symptomatic individual has spread the infection is only half as long into their infection (2 to 3 days). If we can use tracing to test and quarantine or isolate the exposed individual, they in turn will only have spread the disease with 50% probability to someone else[3]. Those individuals in turn will likely be only a day into their infection and so on. Tracing in this fashion, even if it is somewhat imperfect, has the potential to dramatically enhance the leverage we get out of those we identify for testing from the broader population either through random testing or because they are symptomatic.

We hope further analysis in the very near term will refine these estimates. Throughout this note, we treat tracing as if it is a top-down process where "we" identify those with symptoms. However, the most attractive proposals for tracing (Hart et al., 2020) from an ethical viewpoint are actually peer warning systems with little or no top-down component, in which citizens tell each other they may have been exposed and come in for testing. We believe such proposals are highly promising and use the top-down language only as a communicative heuristic and for consistency of discussions of tracing in the epidemiological literature, not to indicate a preference for such solutions.

[3] The word "quarantine" is used in common parlance to refer to the separation from society both of those who have been exposed but who have not yet exhibited symptoms and of those who have symptoms, but in the technical vocabulary of the CDC, those who are exposed but who are not ill or who have been tested and clearly identified as positive or negative are quarantined, while those who have been exposed and are ill or who have tested positive are isolated.

02 Modeling Assumptions

Throughout our analysis we maintain certain key modeling assumptions, though we plan to make available along with this analysis a model that can be easily adjusted to allow the public to plug in their own assumptions on these dimensions.

1. We assume that infections last 15 days from first exposure until spread is no longer occurring, either because of convalesce, an individual being so sick they are isolated anyway, or death (Verity et al. 2020; Zhou et al. 2020; Guan et al. 2020). This is roughly in line with public guidance on periods of self-isolation (CDC 2020) and most individuals sick for 15 days will be self-isolating in any case, and thus observed spread likely concentrates in this 15-day period (Liu et al. 2020).
2. We assume that spread is uniform throughout the infection (a bit more seems to concentrate in the pre-symptomatic stage (Ferretti et al. 2020; He et al. 2020; Nishiura et al. 2020), partly because those with symptoms are more likely to self-isolate in any case, but this simply strengthens our point below). This is somewhat conservative, as it is often the case that spread concentrates during the symptomatic period (Ganyani et al. 2020). We believe conservatism here is appropriate because we have little to no evidence of symptomatic transmission being higher than asymptomatic transmission for COVID-19.
3. We assume that 20% of those infected never develop symptoms (Bi et al. 2020) and yet nonetheless all spread the disease with the same rate as those who eventually develop symptoms. Using 20% for those who are asymptomatic is consistent with the most recent analysis of the Diamond Princess cruise ship (Mizumoto et al. 2020) and while this estimate is still debated, our assumption of asymptomatic individuals spreading at the same rate as symptomatic individuals is quite conservative.

Modeling assumptions

4. We assume that tests never yield false positives, but yield false negatives with a 20% rate (Yang et al. 2020). We assume this rate is unrelated to whether a patient is symptomatic and that retesting will again yield a false negative so that it is never worth retesting a patient who tests negative. This again is quite conservative as it is generally assumed false negatives are more common among asymptomatic individuals and that retesting has some efficacy.
5. We assume that, in the absence of any controls, the disease will spread to 2.5 individuals (Li et al. 2020; Ferretti et al. 2020; Zhang et al. 2020) over the course of a 15-day infection, or roughly once every 6 days.
6. We assume that by the time this policy is enacted, the population prevalence of COVID-19 will be one in a thousand. While this is very high relative to measured rates anywhere in the world at present, it is consistent with expected prevalence rates by early summer in most epidemiological models that include the type of social distancing currently being practiced in most developed countries. Some models indicate that prevalence may be even higher, but this seems a safe number for the prevalence in the population that is circulating (many more may be isolated, but we assume these individuals will not spread the virus).
7. We assume that symptomatic individuals will come in to be tested, but that there will also be individuals presenting for testing who do not actually have COVID-19. We assume that all those with the flu will also present for testing. Flu prevalence during summer is roughly four per thousand, so we assume four people will falsely present for testing for each COVID-19 case.
8. We aim for a target of a typical individual spreading the disease to .75 others during their infection, which ensures a gradual exponential decay of prevalence. Again this is quite conservative as even holding this number to one avoids exponential growth (Ferretti et al. 2020).
9. We discuss further assumptions about tracing technologies in the associated sections below.

03 Untargeted Testing

Suppose we adopt a policy of completely untargeted testing. While this is clearly a naïve policy, it is particularly easy to analyze and some (Romer and Shah 2020) have suggested it might be reasonably effective. In this regime, we do not even allow those who are symptomatic to get greater access to the test. We simply randomly test the entire population in a rotating manner every so many days.

If we adopt this policy and test every x days, with 20% chance that we never catch an infection (because of the false negatives) and with 80% chance that we catch an infection on average x/2 days after it begins, we are just as likely to catch the infection at the very beginning as later. Thus, the individual will pass on, over the course of their infection, the virus to others on average at the following rate.

$$.2 \times 2.5 + .8 \times \frac{2.5 \times x}{2 \times 15} = .5 + .06667x$$

To ensure this number is below .75, we need x below 3.75 and thus need to test the entire population every four days or so. The current US population is 330 million, so this would require testing nearly 90 million people a day.

One remarkable feature of this calculation is that it is *completely independent of the prevalence rate of the disease*. This means controlling the disease through completely random testing is just as hard when prevalence is very low as it is when it is very high. This is an enormous downside of random testing. Furthermore this suggests that coarse forms of targeting, such as by large geographies (like cities) or even by professions, may not be very effective if there is limited mixing in the population across these groups, because unless the disease is controlled in every closely mixing population it will grow, eventually without check, and spread to the rest of the population.

Untargeted Testing

To make matters worse, without further targeting we cannot even take much advantage of people coming in for testing when they have symptoms. In fact, for the parameter values we study, this literally yields no improvement at all in the required testing. Testing everyone (including those who just have the flu) who presents with symptoms does not require a lot of testing capacity. If people present themselves as soon as they become symptomatic, this will be on the fifth day of their infection, and this will occur in the 80% of the population that is symptomatic. This means we'll have to test at the following rate.

$$\frac{.001}{5} \times .8 = .00016$$

This translates to two in 10,000 every day. For each real COVID case we'll also have to test four flu cases, so this brings the number we need to test to one in 1,000 per day, but that still is only 3 million or so per day in the US.

Yet it is important to note this will not be nearly enough to control the disease on its own. To understand this, note that by the time symptomatic patients show up, they will already have infected .833 people. Furthermore, 20% of those infected will be asymptomatic throughout the time they have the disease, and 20% of those tested will yield false negatives[4]. This means that a policy of only testing those who present with symptoms and only quarantining those who test positive will lead an average infected individual to infect others at this rate:

$$.8 \times .8 \times .833 + (1 - .8 \times .8) \times 2.5 = 1.4$$

[4] It is possible we could do somewhat better with a policy of quarantining symptomatic individuals who test negative, though this effect is likely small enough to neglect here and it is far from obvious that this policy is desirable given how many flu cases it would require quarantining for each COVID case caught.

Untargeted Testing

Under this scenario, the rate is well above one and only somewhat slows the exponential growth of the disease.

We thus must supplement this coarse targeting with random testing. Yet if we randomly test more frequently than once every five days, we get literally no value out of people coming in when they are symptomatic because we are testing so frequently that everyone will be caught prior to coming in for testing. So, we must see if randomly testing everyone every five days suffices to control the disease.

Note that if we test every five days, then the contribution to spreading from symptomatics is the same as asymptomatics. We catch everyone on average two and a half days into their infection, except for the false negatives, who we never catch. Thus, an average infected individual spreads the disease to a number of others given by

$$.2 \times 2.5 + .8 \times \frac{2.5 \times 5}{2 \times 15} = .833 > .75$$

We must therefore test more than once every five days, undermining any value of symptomatics coming in themselves. Given that testing symptomatics also generates wasted tests from flu patients, we should in this case simply give up on testing symptomatics and only test randomly.

Our conclusions are substantiated by the more rigorous modeling included in the appendix, which produced estimates of a similar order of magnitude to those discussed above. Both a standard epidemiological model, focusing on infection and recovery rate, and an equilibrium analysis centered on current hospital capacity show that approximately 50 million tests would be needed per day to control the epidemic with random testing, off from our figure by less than a factor of 2. The convergence of these various approaches on similar figures strengthens our confidence in the core conclusions.

Untargeted Testing

One note that may change this conclusion is if asymptomatics and pre-symptomatics spread much less than symptomatics do, as has been suggested (He et al. 2020; Ganyani et al. 2020). In this case, simply quarantining all symptomatics (perhaps even without testing to distinguish their illness from flu) may be effective in controlling the disease. However, this seems quite unlikely, given that such a policy has proven ineffective in every country that has tried it. Likely spread rates are even higher among the very sick, but they are isolated in any case under any policy and thus the assumptions above are roughly correct for the cases considered.

04 Precise Tracing

Now suppose that we have a high-precision contact tracing technology that allows us, by tracing the five contacts per day since infection, to find a sufficiently high fraction of those infected to control the virus (as took place in South Korea without blanket, mandatory lockdown). Suppose that in addition to these five contacts, again roughly as in Korea, we must trace and test five close friends a day, as well as all those presenting with symptoms or false flu symptoms. Suppose that, as in the Korean case, we manage to track down 75% of cases from tracing and the remaining enter the system via symptoms. A couple of possible technologies that would allow this are traditional/manual contact tracing assisted by digital aides-de-memoir (such as contact lists, personal location data, etc.) or the Bluetooth-assisted, high-precision private contact tracing pioneered in Singapore and adapted to better preserve privacy by many ongoing projects in the US and Europe (Hart et al., 2020). Note that given the technical challenges in making the Bluetooth technology work, we would need near-universal adoption of this technology, at least in dense urban areas, to achieve this with Bluetooth alone, as well as quite a bit of luck, given the current state of imperfectly understood technical issues.

How much testing would it take to sustain such a strategy in the context of the US with the relevant rates of flu and COVID-19? Because there are 20% of cases we will miss due to false negatives in this approach, we must keep the average spreading among the rest to .25, which occurs after a tenth of the course or 1.5 days into the disease, on average. During those 1.5 days, a typical individual will have had 7.5 contacts, plus 5 friends or close family members for a total of 12.5. To achieve this level of testing, we must be catching on average .067% of the population every day, or approximately 230,000 people per day. With 75% of those being tested coming from contact tracing, which requires testing 12.5 contacts of each case caught, the other 25% would come from testing symptomatics, which, as we saw above, requires testing four false cases for every real one. Thus, overall, we need approximately 11 tests for each case caught, or a total of 2.5 million tests per day. This also reflects the lower bound of 1 to 2 million tests per day arrived at in the appendix through two models, both assuming tracing capacity reaches the point where a person tested is 30 times more likely to be infected than a randomly selected member of the general population.

05 Imprecise Tracing

Another, less precise tracing approach was used in Taiwan. Those infected would report their location history to a repository, and users could cross-check their own location history against this repository and request a test if there was an intersection. Given the imprecision of GPS technology, this led to a huge false positive rate. It too roughly 100 tests per day of circulation prior to the infection being caught to trace all relevant contacts. On the other hand, this had the advantage of catch 90% of all transmissions. In the US, this is called the GPS Heatmap approach (Hart et al., 2020).

Repeating our calculation for precise tracking using these parameters, this requires us to administer roughly 140 tests for each case caught or about 30 million tests per day.

06 Cost Estimates

How much does all this cost? First let's focus on the cost of testing. Costs of producing tests are falling rapidly, and the tests are also becoming easier to administer. Given this, and the fact that present tests cost approximately $30 to $50 per test (CMS 2020), we estimate approximately $15 per test. Even this may be an overstatement, given that much of the labor that goes into producing and administering the tests might otherwise be idle given the depression already caused by COVID-19.

If we take the $15 figure, the cost of administering 1 million tests per day for 12 months of pandemic beginning after ramp up (until a vaccine is available) is approximately $5 billion. This means that to administer 5 million tests a day over this duration is $25 billion; this gives us a lower bound of the cost of the testing regime if we can use targeting extremely effectively. In the worst case of untargeted or poorly targeted testing, we should instead expect a cost of $500 billion or more. In the intermediate case of tens of millions of tests, we should expect a cost in the high tens or low hundreds of billions of dollars.

We need to add to this the costs of contact tracing. If most contact tracing is digitally mediated, the financial costs are low, though costs along other dimensions of privacy or civil liberties may be higher depending on the design. If we must hire manual contact tracers—and we count these hires as costs, rather than opportunities to employ the unemployed—imagine that we pay a typical contact tracer $50k a year. And suppose a typical contact tracer can handle two cases in a day, which is about the rate achieved in Australia with limited training. With a prevalence rate of .1% of the population and with 2/3 of cases caught after two days and traced, this means tracing a few hundred thousand cases a day, which would require roughly 100k tracers. The salary of these tracers for this year would then be about $5 billion, a small fraction of the cost of the testing infrastructure to begin with.

Cost Estimates

This suggests a best-case cost of the program of $30 billion, a worst-case cost of $500 billion, and a typical cost of high tens or low hundreds of billions of dollars. Note that this compares to estimated costs of more than even this worst-case estimate for each month of continued uniform lockdown.

07 Conclusion

For somewhere in the ballpark of tens to hundreds of billions of dollars, combined with an intelligent used of tracing, we can end a lockdown that is costing the US economy tens of billions of dollars every day. If we instead target the much smaller number of tests suggested in the AEI and CAP analyses, we cannot safely leave lockdown. Failing to make this investment would go down as one of the most extreme examples in history of being pennywise and pound foolish. A key impediment to scaling up the supply chain is the lack of demand and supply perceived at every step of the testing supply chain. Achieving common acceptance of the need for tens of millions of tests a day and coordinating efforts to hit this target is therefore critical to our ability to go outside again. We must communicate this message as clearly and as loudly as we can to as many leaders as possible, and as quickly as possible.

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09 Appendix

Estimates of Required COVID-19 Testing for the US

April 7, 2020

1 Introduction

We are interested in how many people the US will need to test per day in order to control the COVID-19 pandemic. We present three estimates. The first is a simple extension to the standard susceptible-infected-recovered model used in the epidemiological literature. The second is an equilibrium analysis that derives the number of tests required to keep the number of new hospitalizations from exceeding the number of people recovering from hospitalization. The final approach uses the experience of Taiwan and South Korea to back out a best-case number of tests per day for the US. Even assuming targeted testing, our main conclusion is that the US will need on the order of millions of tests per day to control the spread of COVID-19. Table 1 provides a breakdown of testing implications for each model.

An important caveat: none of the authors is an epidemiologist and we believe a more complex and realistic model developed by experts in the field will yield a more accurate estimate. Our goal is provide a simple baseline that we hope captures the first order testing requirements. We also hope that once testing becomes widespread, we will get a better handle of the true infection rate and so will be able to calibrate the model more accurately.

Model	**Required Tests per Day under Targeted Testing**
Targeted SIR Model	$1 - 10$ million
Equilibrium Analysis	~ 4 million
Taiwan and South Korea Case Study	~ 3 million

Table 1: Implied testing requirements by model

2 SIR Model with Targeted Testing

2.1 Model Setup

We build on the standard susceptible-infected-recovered model used in the epidemiological literature. Specifically, we allow for testing and a quarantined population. Our model is governed by the following set of differential equations:

$$\dot{S} = -\beta \times S \times \frac{I}{N} \tag{1}$$

$$\dot{I} = \beta \times S \times \frac{I}{N} - N_{test} \times f \times \texttt{posrate}\frac{I}{N} - \gamma \times I \tag{2}$$

$$\dot{Q} = N_{test} \times f \times \texttt{posrate} \times \frac{I}{N} - \gamma \times Q \tag{3}$$

$$\dot{R} = \gamma \times (I + Q) \tag{4}$$

$$H = \text{hosprate} \times (I + Q) \tag{5}$$

where:

- $I(t)$ is the number of infected people who are not in quarantine at a given point in time
- R(t) is the number recovered or deceased people
- S(t) is the number of individuals susceptible to being infected
- Q(t) is the number of individuals in quarantine
- H(t) is the number of hospitalized individuals
- N is the total population
- β is the number of people an infected person infects per day
- γ is the fraction of infected people that recover per day
- f is targeting efficiency defined as the ratio of the probability that the population given the test have the virus vs. general population
- posrate is the probability test gives positive result, given person has virus
- hosprate is the fraction of infected who require hospitalization
- N_{test} is the number of people to be tested per day

To simulate the model, we begin with a set of initial conditions and then integrate the differential equations above.

2.2 Calibration

In order to get an order of magnitude sense for how many tests the US will need to control COVID-19, we plug in reasonable parameter values and simulate the model. Our parameter choices are:

- $I(0)$ = N/1000 is the number of infected people who are not in quarantine at the
- beginning of the simulation
- $R(0)$ = 0 is the number recovered at the beginning of the simulation
- QO)= 0 is the number of individuals in quarantine at the beginning of the simulation
- $S(0)$ = N - I(0) is the number of individuals susceptible to being infected
- N = 330 million is the total population
- β = 1/6 number of people an infected person infects per day
- γ =1/12 mean recovery rate of infected
- f = 1, 10, 30 ratio of fraction of people administered a test who have the virus with respect to the fraction of people in the general population who have the virus
- $\mathrm{posrate}$ = 0.80 probability that the test gives a positive result, given a person who has the virus
- $\mathrm{hosprate}$ = 0.20 fraction of infected who require hospitalization

One of the more important parameters is the extent to which the testing is targeted (f). Recall that f is the ratio of probability that people given a test have the virus vs. general population. A value of f = 1 corresponds to the case where we simply randomly sample from the population as a whole. We refer to this as "naive testing." A value of f = 30 corresponds to the case where people who are tested have 30 times higher probability of being infected than general population.

Appendix

2.3 Simulation

We solve the model under different testing assumptions and show the result in Figure 1. We see that the size of the hospitalized population falls off with increased testing, as expected. Further, the number of tests required under naive testing strategies (low f) is dramatically higher than for more targeted testing (high f). Precise values of f are likely to vary by geography and over time, but this plot shows that keeping them as high as possible is important. For approximate values of f (between 3 and 30), we see that between 1 and 10 million tests/day will be required in order to keep the peak hospital population below:

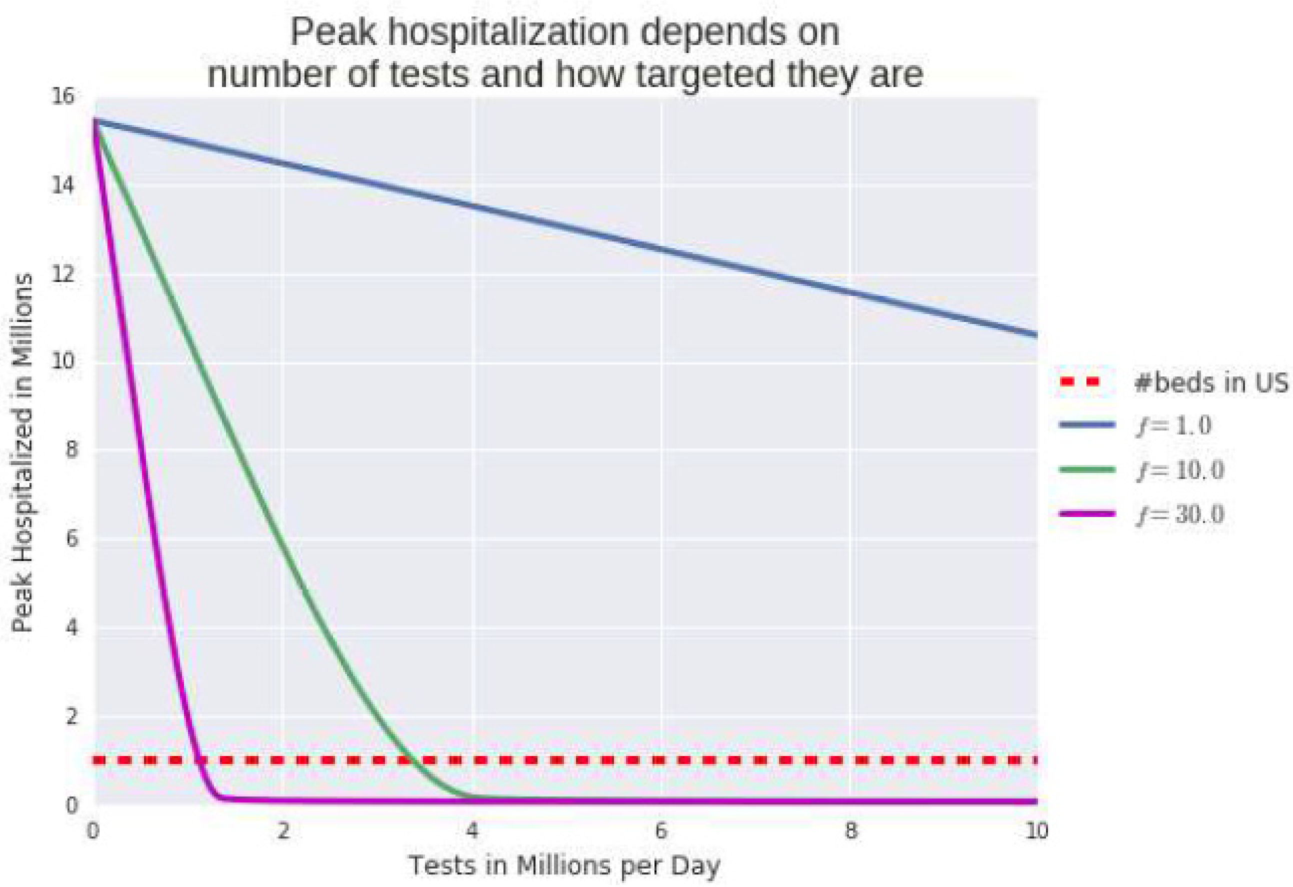

Figure 1: Maximum fraction hospitalized as a function of number of tests

Appendix

2.3 Simulation continued

In this figure, we summarize the outcome of our model. The various lines show the number of hospital beds required to treat COVID patients under various different targeting efficiencies f (larger is more targeted) as we increase the number of tests that can be per formed per day. The total number of hospital beds is included as a dotted line; of course, the number of beds available for COVID patients is a subset of all beds. From this plot, we can read off our headline numbers: for reasonably efficient targeting, f around 10 to 30, a few millions of tests per day will be needed to keep total hospitalizations below the maximum.

1 million. Although high f is desired, tests should also be spread out as much as practical while keeping a high f value - oversampling a single cluster will result in unchecked growth
in other regions and degrade the advantages provided by the testing.

3 Testing Requirement Based on Equilibrium Analysis

3.1 Summary

We derive the minimum number of daily tests required to ensure that hospitals will be able to stay within capacity throughout the COVID-19 crisis. We consider two equilibria:

1. the largest possible size of infections within the population at which hospitals can still function
2. the required number of daily tests to ensure that the number of infections does not grow beyond this limit.

The main takeaway is that the number of tests estimated by this method is **at least 41 million per day with random testing** or **at least 4.1 million per day with targeted testing** (for test efficiency $f = 10$). The number of required daily tests can be expressed as:

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$$\text{Number of Daily Tests needed} \geq \text{Size Of Population} \times \frac{1}{f} \times \frac{1}{\text{posrate}} \times \tag{6}$$

$$(\text{rate of spread} - \text{rate of recovery/removal}) \tag{7}$$

- Size Of Population = size of the US population, assumed to be 330 million
- rate of spread = how many people a spreader (=infected and not quarantined person) infects per day, assumed to be 1/6 because every 6 days one new person gets infected
- rate of recovery/removal = fraction of spreaders that recover or die per day, assumed to be 1/15 as it takes 15 days to recover/die
- f = target efficiency = ratio of probability that people given test have virus vs. general population
- posrate = probability test gives positive result, given person has virus, assumed to be 80%.

Dependence on targeting efficiency:

Targeting efficiency	1	10	30
Daily tests needed	41.3Mn	4.1Mn	1.4Mn

3.2 Estimation

We need to hold the total number of new infections per day below a certain threshold or we won't have enough beds in hospitals. The number of new beds needed, *N*needed, is the number of newly infected *Ni.rs* times the hospitalization rate, *h*. Here *N*; is the number of currently infected people, and rs is the number of people who would be infected for every **person** currently infected. The number of beds freed is the total number of beds Btotal times the rate of recovery/removal rr (we include deaths here to keep notation simple). Therefore, we have

Appendix

3.2 Estimation continued

$$B_{\text{needed}} = B_{\text{free}} \quad \Rightarrow \quad N_i \times r_s \times h = B_{\text{total}} \times r_r \tag{8}$$

This relation can be used to give a bound on the number of currently infected people, at which hospitals run at maximum capacity

$$N_i \leq \frac{B_{\text{total}} \times r_r}{r_s \times h} \tag{9}$$

When equality holds in equation 9 we call the above condition the **critical equilibrium of maximum hospital capacity.**

Now we study the condition under which the number of spreaders will stay at the critical equilibrium (equation 9). Crucial here is a **large enough number of tests**. Recall that $Ns(t)$ is the number of spreaders on day t. The number of *spreaders* $Ns(t=1)$ on the next day is given by

$$\begin{aligned} N_s(t+1) = & (1 - \texttt{rate of recovery}) \times N_s(t) + (\texttt{rate of spread}) \times N_s(t) \\ & - (\texttt{number Of Daily Tests}) \times \frac{1}{f} \times \frac{1}{\texttt{posrate}} \times \frac{N_s(t)}{\texttt{Size Of Population}} \end{aligned} \tag{10}$$

It is composed of 3 parts (corresponding to the 3 summands in the above expression):

1. Number of spreaders that are still infected: (1 – rate of recovery) $N_S(t)$
2. Number of newly infected people: (rate of spread rate) $N_S(t)$
3. Number of infected people that are are removed from the population of spreaders because they have been diagnosed by testing: f = test efficiency, assumed 1 for random testing, 10 for targeted testing, posrate = probability test gives positive result, given person has virus, assumed to be 80%.

Appendix

3.2 Estimation continued

In order for the number of spreaders to not grow (and in particular not grow at the critical threshold (equation 9), we need $N(t+1) < N(t)$. Using equation 10, the inequality becomes the **minimum required testing condition**

$$\text{Number of Daily Tests needed} \geq \text{Size Of Population} \times \frac{1}{f} \times \frac{1}{\text{posrate}} \times (\text{rate of spread} - \text{rate of recovery}) \tag{11}$$

4. Minimum Tests for Partial Social Distancing: the Case of Asian Countries

4.1 Introduction and Summary

Several Asian nations have managed to control the spread of COVID-19 without fully going to stay-at-home orders. We take Taiwan and South Korea as models of this approach. We try to extrapolate some lower bound on the level of testing needed in the US from the level of testing used in South Korea and Taiwan.

To complement the structural modeling approaches outlined elsewhere in this **white paper**, we can take the South Korean/Taiwan (hereafter, SK/T) numbers and scale them to the scale of the problem in the United States. This approach can give us an **optimistic minimum number** of tests needed for a country the size of the US with cases on the scale of the US. This approach does not try to account for all the differences between the US and SK/T and assumes that such details will tend to average out in the final estimate. We expect some of the particularities of the US to push the estimate up, while other particularities (lower density) to push the estimate down. Finally, we emphasize that this **is an estimate assuming that there is still partial social distancing**. Return to “normalcy” will necessarily require more tests than this, perhaps by an order of magnitude or more.

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4. Minimum Tests for Partial Social Distancing: the Case of Asian Countries continued

In the sections below we estimate that based on the level of testing in SK/T the US would need several million tests per day,

$$R_{\text{USA}} \simeq \texttt{3Mn tests /day} \tag{12}$$

This number should be understood in the context of the current level of testing in the USA of about 100,000 per day.

4.2 Basic Data Used

As of April 6th, the number of confirmed cases in SK is 10,248, while the number of tests conducted are 443,000 to date. The testing was done over a 40-day period with essentially 10,000 tests a day.

https://www.statista.com/statistics/1102777/south-korea-covid-19-daily-new-cases/

https://ourworldindata.org/covid-testing

The situation in TW is a bit less clear, but appears to be about 400 cases and 20,000 tests. The time scale over which these tests were conducted is not clear.

https://focustaiwan.tw/society/202003240013

Currently there are 300,000 known cases in the USA, and about 1.27 million people tested.

4.3 A Naive Scaling Argument

It seems plausible that for effective mitigation the number of tests needed is roughly pro portional to the number of known cases:

$$N_{\text{test}} = \alpha N_{\text{known}}$$

Appendix

4. Minimum Tests for Partial Social Distancing: the Case of Asian Countries continued

Based on South Korea (10,000 cases with roughly 400,000 tests) and Taiwan (400 cases with roughly 20,000 tests) we estimate that α ~ 50. This seems reasonable and can be interpreted as the average number of people any of the N_{known} cases could have interacted with. Note that the total size of the population does not play a role here as we assume that some sort of localization of the problem is still feasible (contact tracking").

For the US: $N_{known} = 300,000$

Which would suggest that a total of, $N_{test} = 15\text{Mn} \quad (\alpha \sim 50)$

This is the total number of tests we would need to perform assuming that we start today with 300,000 known cases and a total of 1 million tests already taken (which we neglect as it is much smaller than the number needed).

To translate this number to the number of tests needed per day, we need some notion of time-scale in the problem. One natural time-scale is the doubling time, namely how long would it take before the number of known cases doubles (go from 300k to 600k in the US). This number is currently estimated at τ = 5 days for the US. A testing scheme that would take much longer than this would hardly have any relevance since the number of cases would be exponentially larger by the time it is concluded. So an order of magnitude estimate for the number of tests per day to come anywhere close to dealing with the problem is:

$$R_{\text{USA}} = \frac{N_{test}}{\tau} = \frac{15,000,000}{5} \simeq 3\text{Mn / day}$$